Name _____   Summer 2022
Bode/Kahan/Yoo

> **Calculus Summer Assignment 2022**
> **No Calculator Allowed**
> **To Be Completed by: Tuesday, Sept.6, 2022**

The problems in this assignment are to be completed without the use of a calculator of any kind.
Show all of your work and clearly indicate final answers; **any form of a correct answer will receive full credit**. You may use your notes from Precalculus, texbooks, online sources or a peer as needed. Your work must be completed by the date above, as your first exam will include questions on the topics included in this assignment.

These topics are:
1. Limits
    a) by substitution
    b) using algebraic simplification then substitution
    c) one-sided limits and piecewise functions
    d) around a vertical asymptote
    e) trigonometric limits
    f) as x approaches $\pm\infty$
    g) graphically

2. Derivatives
    a) Definition of the Derivative
    b) Power Rule
    c) Equation of a tangent line
    d) Horizontal tangent line
    e) Product rule
    f) Quotient rule
    g) Trigonometric Functions
    h) Chain Rule

3. Continuity of a Function
    a) Definition of Continuity
    b) Types of discontinuities – P.O.D., Vertical Asymptote, Jump

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I. Calculate each limit:

1. $\displaystyle\lim_{x\to 0}\frac{x-1}{x^2-1}$

2. $\displaystyle\lim_{x\to 3}\frac{x-3}{\dfrac{1}{x}-\dfrac{1}{3}}$

I. Calculate each limit (cont.)

3. $\lim\limits_{x \to 9} \dfrac{\sqrt{x}-3}{x-9}$

4. $\lim\limits_{x \to 0} \dfrac{(x-1)^2-1}{x}$

Remember: $\lim\limits_{x \to a} f(x)$ **exists if and only if** $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x)$.

5. $\lim\limits_{x \to 3} f(x)$ if $f(x) = \dfrac{x}{x-3}$

6. $\lim\limits_{x \to 3} g(x)$ if $g(x) = \dfrac{x}{(x-3)^2}$

7. Let $f(x) = \begin{cases} x^2 + 3, & x \le -2 \\ 5 - x, & x > -2 \end{cases}$

   Find: a) $\lim\limits_{x \to -2^-} f(x)$ b) $\lim\limits_{x \to -2^+} f(x)$ c) $\lim\limits_{x \to -2} f(x)$

I. Calculate each limit (cont.)

8. Let $f(x) = \begin{cases} x+2, & x < 0 \\ \sqrt{x} + 2, & 0 \le x < 1 \\ \ln x, & x \ge 1 \end{cases}$

Find: a) $f(0)$ b) $\lim\limits_{x \to 0} f(x)$

   c) $f(1)$ d) $\lim\limits_{x \to 1} f(x)$

   e) Is $f(x)$ continuous at $x = 0$? Justify your answer.

   f) Is $f(x)$ continuous at $x = 1$? Justify your answer.
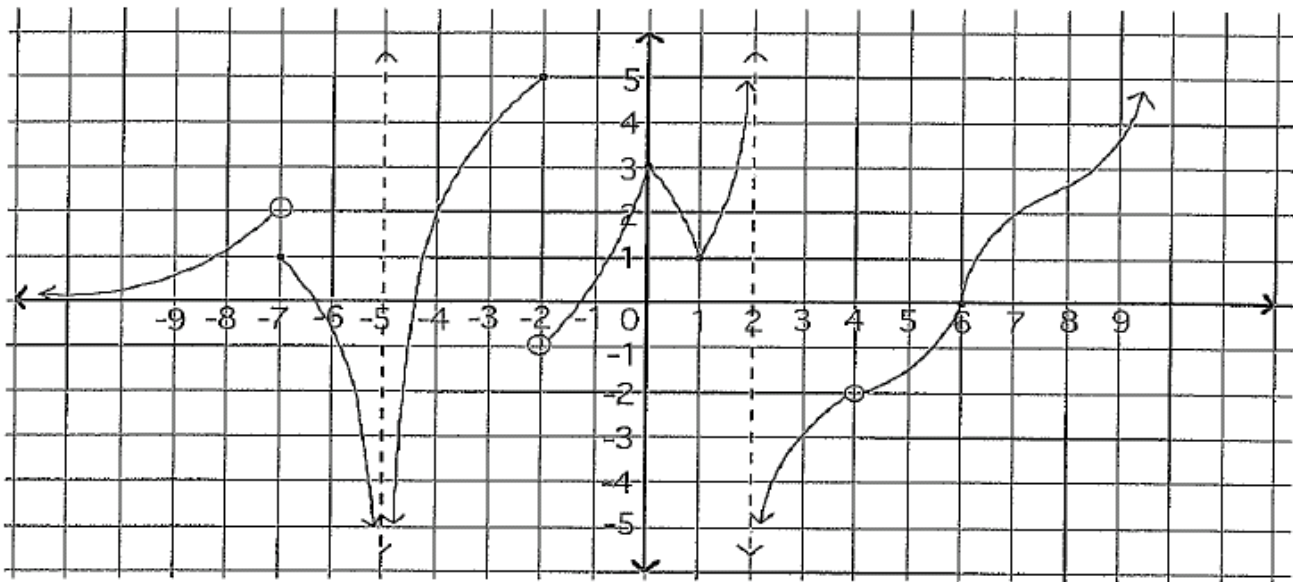
Some special trigonometric limits

9. $\lim\limits_{x \to 0} \dfrac{\sin x}{x}$

10. $\lim\limits_{x \to 0} \dfrac{5\sin x}{4x}$

11. $\lim\limits_{x \to 0} \dfrac{\sin x - x}{3x}$

12. $\lim\limits_{x \to 0} \dfrac{\cos x \tan x}{x}$

Calculate each limit based on the graph of $f(x)$ **is shown below:**

13 a) $\displaystyle\lim_{x\to-2^-} f(x)$

b) $\displaystyle\lim_{x\to-2^+} f(x)$

c) $\displaystyle\lim_{x\to-2} f(x)$

d) $\displaystyle\lim_{x\to 4^-} f(x)$

e) $\displaystyle\lim_{x\to 4^+} f(x)$

f) $\displaystyle\lim_{x\to 4} f(x)$

g) $\displaystyle\lim_{x\to 2^-} f(x)$

h) $\displaystyle\lim_{x\to 2^+} f(x)$

i) $\displaystyle\lim_{x\to 2} f(x)$

j) $\displaystyle\lim_{x\to-7} f(x)$

k) $\displaystyle\lim_{x\to-5} f(x)$

l) $\displaystyle\lim_{x\to 1} f(x)$

m) $\displaystyle\lim_{x\to-\infty} f(x)$

n) $\displaystyle\lim_{x\to\infty} f(x)$

p) State each value of x at which $f(x)$ is not continuous and state the type of discontinuity.

14. $\lim\limits_{x\to\infty} \dfrac{1-x^2-x^4}{3x^4+1}$

15. $\lim\limits_{x\to\infty} \dfrac{1-x^2-x^3}{3x^4+1}$

16. $\lim\limits_{x\to\infty} \dfrac{1-x^2-x^4}{3x^3+1}$

17. $\lim\limits_{x\to-\infty} \dfrac{1-x^2-x^4}{3x^3+1}$

II. Derivatives

Limits involving the definition of the derivative.

18. $\lim\limits_{h\to 0} \dfrac{\tan(x+h)-\tan(x)}{h}$

19. $\lim\limits_{h\to 0} \dfrac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$

Derivatives Power Rule

Find y' for each:

20. $y = 3$

21. $y = -3x^2 + 5x - 7$

22. $y = \sqrt{x} - \dfrac{5}{\sqrt{x}} + \dfrac{3}{x^3}$

Derivatives Product Rule

23. Find $\dfrac{dy}{dx}$ using the product rule. **Verify by distributing first then finding $\dfrac{dy}{dx}$ if $y = (x^2 + 3x)(x + 5)$.**

24. Find $\dfrac{dy}{dx}$ if $y = x^2 \sin x$

Derivatives Quotient Rule

Find y' for each:

25. $y = \dfrac{3x - 2}{2x - 3}$

26. $y = \dfrac{x + 1}{\sqrt{x}}$

Derivatives Trigonometric Functions

Find y' for each:

27. $y = \sin(x)\cot(x)$

28. $y = \sec(x)\tan(x)$

29. $y = \dfrac{\cos(x)}{\sec(x)}$

Derivatives Using the Chain Rule

Find y' for each:

30. $y = (4x^2 + 1)^3$

31. $y = \cos(x^2)$

32. $y = \sin^3(\cos(x^2))$

33. $y = \sqrt{x + \tan x}$

Tangent Line Equations

34. Find the slope of the normal line of $f(x) = \dfrac{x}{x-1}$ at $(2, 2)$.

35. Write an equation of the tangent line of $f(x) = -3x^2 + 5x - 7$ at $x = 1$.

36. State the coordinates of each point(s) at which the graph of the equation $y = \dfrac{x^3}{3} - \dfrac{3x^2}{2} + 2x$ has horizontal tangent line(s).

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37. Given $f(x) = \dfrac{x^2 + x - 2}{x^2 + 3x - 4}$, find:

a) the coordinates of all points of discontinuity,

b) an equation of all vertical asymptotes and

c) an equation of the horizontal asymptotes.

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38. Using your knowledge of logarithms answer the following

a) Write as a single ln: $\dfrac{1}{2}\ln(x) + \ln(x+1) - 3\ln(x^2 + 4)$

b) Solve for t; leave your answer in terms of ln: $100 = 50e^{3t}$

c) Find the value of each logarithm:

    (i) $\ln e$ (ii) $\ln 1$

    (iii) $\ln e^3$ (iv) $\ln \sqrt[3]{e}$